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THE DEPENDENCE OF THE INDUCED EFFECT ON ORIENTATION AND A HYPOTHESIS CONCERNING DISPARITY COMPUTATIONS IN GENERAL

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Abstract—The induced size effect is an apparent rotation about a vertical axis that results from binocularly viewing a target in which one half-image is vertically magnified. A previous paper (Arditi *et al.*, 1981, *Vision Res.* **21**, 755–764) described a theory of this effect in terms of *horizontal* disparities that exist between vertically magnified images of oblique features and their unmagnified counterparts. The present studies test two aspects of that theory: the requirement of oblique features in stimuli eliciting the induced effect, and the assumption that binocular associations (inputs to disparity computations) are made across horizontal meridians. The former aspect was confirmed in a stereo discrimination experiment in which the direction of rotation (tilt) for crossed line patterns of varying orientation was judged, for a fixed vertical magnification of one half-image. The latter aspect was rejected on the basis of the results of that experiment, and of two experiments in which observers matched the apparent tilt of the lines with a horizontal adjustment line which could be stereoscopically rotated in depth. The data and some associated demonstrations suggest that stereoacuity and apparent depth of oblique lines vertically magnified in one half-image are determined by the horizontal separation between binocular points which are *nearest* in a fixed binocular coordinate map, rather than by purely horizontal point-matchings. This "nearest neighbor hypothesis" seems to be operative in classic measures of stereoacuity as well as in the induced effect.

INTRODUCTION

Some forty years ago, Kenneth Ogle published several papers dealing with the distortions of stereoscopic space which occur when one introduces a weak meridional (e.g. afocal or cylindrical) lens over one eye under binocular viewing conditions (Ogle, 1938). For example, the introduction of a weak lens which increases the horizontal magnification of the right eye will cause an objectively fronto-parallel plane to appear as if the right side of the field is more distant than the left side of the field. Figure 1 illustrates how

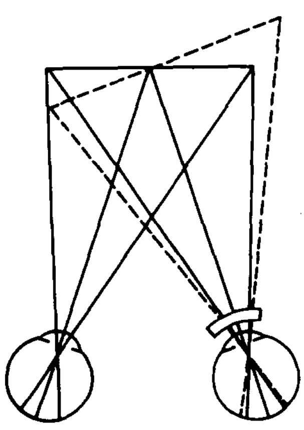

Fig. 1. Diagram showing the apparent tilt of a frontoparallel surface produced by a meridional magnifying lens oriented axis vertical (i.e. horizontal magnification) over the right eye.

this phenomenon, which Ogle termed the "geometric effect", relates to the binocular parallax which exists when a plane is objectively tilted from fronto-parallel. If, however, the lens is oriented so that its meridian of magnification is vertical, the plane appears to be of the opposite tilt; that is, the right side of the field appears closer to the observer than the left side of the field. Ogle called this phenomenon the "induced size effect". Although increased vertical magnification of one eye's view is not encountered in normal binocular viewing conditions, it has been widely held that the induced effect is an anomalous stereoscopic response to vertical disparities (Ogle, 1950; Julesz, 1971; Nelson, 1977; Sheedy and Fry, 1979). "Induced effect" is a rather model-bound name, reflecting a possible explanation for the phenomenon: that vertical expansion of one eye's image effectively *induces* horizontal expansion of the other eye's image prior to the stereoscopic process.

Recently, Arditi *et al.* (1981) advanced a theory of the induced effect which involves neither responses to vertical disparities nor any induced size changes. The fundamental geometrical relationships on which this theory rests are depicted in Fig. 2. This shows an oblique line segment, and its vertically magnified counterpart which, as a result of the magnification, is slightly rotated and elongated. Thus if these lines were presented in dichoptic view, a gradient of vertical disparity would exist, increasing from the origin to a maximum of v' at the endpoints of the magnified and unmagnified lines. Less obvious is a gradient of *horizontal* disparity, increasing from the origin to a maxi-

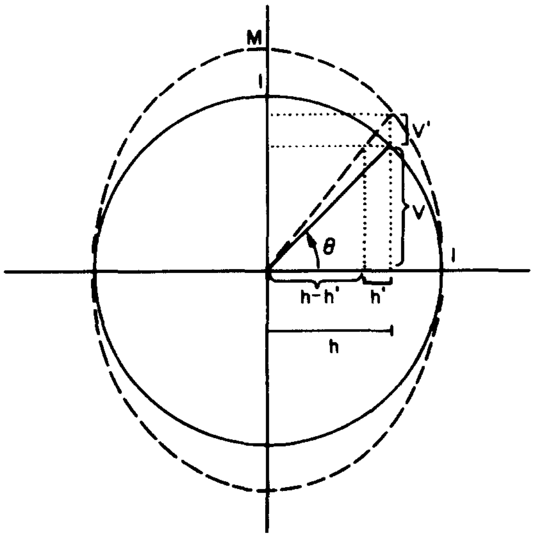

Fig. 2. Diagram illustrating disparities which arise from vertical magnification of an oblique line by factor M. Lengths v and h are vertical and horizontal extents of the unmagnified line; v' and h' are maximum vertical and horizontal disparities between the unmagnified (——) and magnified (- - -) lines; θ is the elevation of the unmagnified line.

mum of h' at the endpoint of the unmagnified line. This horizontal disparity gradient, computed along horizontal meridians, is of opposite sign and nearly equal absolute magnitude to that which would be produced by an equal horizontal magnification.

Thus, if the visual system were to compute depth by associating points on the lines across horizontal meridians, the horizontal disparity gradient produced by vertical magnification of one half-image would both qualitatively and quantitatively account for the induced effect seen when oblique lines or contours are viewed. This idea is referred to here as the "horizontal meridian hypothesis." Stereograms composed of oblique lines showing the induced and geometric effects are shown in Fig. 3a and b, respectively.

The primary concerns of this paper are, first, to determine if the presence of obliques is a necessary condition for stereo discrimination when lines are vertically magnified in one half image, and, second, to explore in detail the effect of varying orientation on both stereo discrimination and apparent tilt. Since the variable of interest in these studies is not orientation but rather the difference in orientation between the test stimulus and the horizontal, this angle, termed "elevation," will be used hereafter.

If the visual system associates points along horizontal meridians, discriminability should be a monotonically decreasing function of elevation of the unmagnified line (θ in Fig. 2), since such horizontal disparities grow with decreasing elevation. One experiment reported here, however, shows that discriminability for both induced effect and geometric effect stimuli is not a monotonic function of elevation, but rather is an inverted U-shaped function. The most parsimonious explanation for the data is that stereoacuity is determined not by the horizontal separation of points across horizontal meridians, but rather by the horizontal separation of *nearest* points in a fixed binocular coordinate map. This idea is termed the "nearest neighbor hypothesis," and is supported by classic measures of stereoacuity as well as the measures used in this study.

Another prediction of the horizontal meridian hypothesis is that perceived tilt in depth should be constant regardless of the elevation (proven in the earlier paper). Other experiments and demonstrations reported here show that for vertical magnification of one half-image, perceived tilt depends strongly on line elevation. Again, the most parsimonious explanation is that disparities between nearest points determine the depth. But for stimuli horizontally magnified in one eye, the horizontal meridian model seems to determine the apparent tilt.

In many conventional stereoscopic stimuli, of course, the "nearest neighbor" disparity is equal to the disparity measured across horizontal meridians. When elevation (orientation) is varied, however, the two types of point matchings are different. This paper purports to show that in many circumstances, nearest neighbor disparities are better predictors of stereoscopic responses than are disparities measured across horizontal meridians.

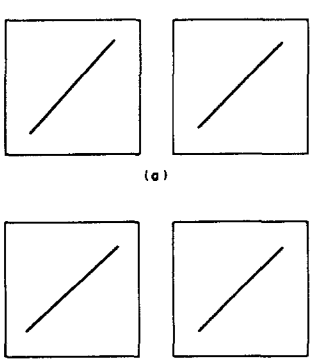

Fig. 3. Oblique line stereogram demonstrating (a) the induced effect, and (b) the geometric effect. The right half-images are magnified by 10%.

METHODS

The experiments were run under the control of a PDP-11/34 computer equipped with a laboratory interface (Cambridge Electron Design LIS 502). The stimulus lines, as illustrated in Fig. 4, were generated as Lissajou patterns in in the following manner: identical triangular waveforms, provided by a function generator, were passed through a pair of independently programmable multiplying digital-to-analog converters (MULDACs) to the X and Y axes of two Tektronix model 5103 oscilloscopes (P31 phosphor). The orientations of the lines were controlled by varying the relative amplitudes of the X and Y oscilloscope inputs through the MULDACs, while the

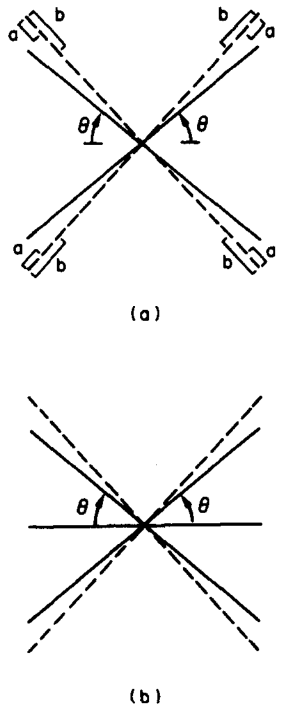

Fig. 4. Stimulus configurations for (a) Experiment 1 and (b) Experiment 2.

lengths of the lines were controlled by varying the absolute amplitudes of these inputs. The individual lines in the stimulus pattern were generated alternately, with appropriate gating signals, in intervals of 0.5 msec. Thus the entire display was refreshed every 2 msec for the pattern in Fig. 4a and every 3 msec for the pattern in Fig. 4b.

The oscilloscope screens were placed at right angles to one another and their images superimposed with a partially-silvered mirror. Binocular separation was achieved by mounting orthogonally oriented polarizing filters on each oscilloscope face and at the observers eyes. The optical distance to each oscilloscope was 430 mm. The unmagnified crossed lines in all experiments subtended 6.9 deg, while the horizontal adjustment lines used in Experiments 2 and 3 subtended 5.4 deg at the beginning of each trial (before adjustment). Observers viewed the display in a darkened room, with head in a chin cup and forehead placed firmly against a rest. Although the shadowed outlines of the apparatus were dimly visible, there was no other stable frame of reference (except for the matching lines in Experiments 2 and 3) available against which to compare the direction of tilt. In the discrimination experiment, the observer both initiated trials and conveyed responses by pressing one of two buttons. This removed the possibility of any apparent motion cues to the direction of depth, since the interval between trials was greater than approx. 0.5 sec.

All results were confirmed using both naive and sophisticated observers; of those whose data are reported here, only the author had previous experience in psychophysical experiments. All observers had normal or corrected-to-normal vision.

EXPERIMENT 1

In this experiment, tilt discrimination was tested for crossed line patterns as shown in Fig. 4a, as a function of θ. The observers were instructed to choose between the alternatives "left in front" or "right in front", when presented with patterns differing between the eyes in three ways: (1) in the "induced effect" or IE condition, one half-image was vertically magnified by a factor of 1.08 with respect to the other. (2) In the "geometric effect" or GE condition, one half-image was minified horizontally with respect to the other half-image by the same factor as a vertical magnification in the IE condition (equivalently, it was horizontally magnified by the reciprocal of the vertical magnification in the IE condition). Thus conditions IE and GE differed only in the presence or absence of the segments labeled b in Fig. 4a. Finally, the "equal length" or EL condition differed from the IE condition only in that the segment a was removed. Thus the three conditions differed only in the lengths of the line in one eye's view. When $\theta = 90$ deg, a "left-in-front", "right-in-front" decision is impossible. Therefore, for these stimuli, the observers were asked to press the "right-in-front" button if the upper portion of the stimulus line appeared closer than the lower, and the "left-in-front" button if the opposite direction of depth was apparent. This condition was included in order to determine whether obliquity is necessary for the induced effect.

The order of presentation was random, and all conditions were counterbalanced for eye of transformation (e.g. vertical magnification). The observers were allowed as much time as needed to make judgements, and were instructed to keep both eyes open while viewing the stimuli. No feedback was given.

Results and discussion

Figure 5 shows the probability of a correct response as a function of the elevation θ for two observers. The upper bound of the 90 per cent confidence band around 0.5 for the normal approximation to the binomial distribution is 0.63; thus only values which fall above this level represent performance greater than chance at a 0.05 level of (one-tailed) significance.

First consider the IE condition alone. When the lines were oriented close to horizontal ($\theta = 2.5$), none of the observers successfully discriminated the direction of depth. As θ increases, however, discriminability improves, until the lines are nearly vertical, where performance again drops to chance. This results alone indicates that vertical magnification of one half-image of a line stereogram requires obliquity of the lines, in order to produce the induced effect.

Now consider the GE and EL conditions, which differ from the IE condition only in the length of the

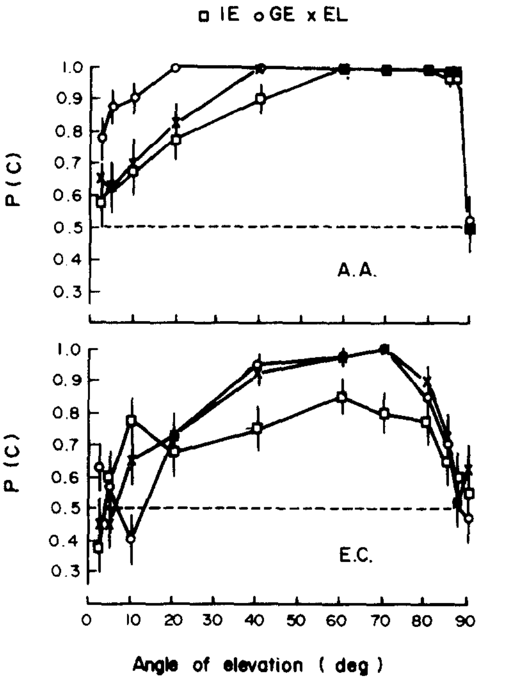

Fig. 5. Probability of discriminating direction of tilt for the oblique line patterns of Experiment 1, for 2 observers, under conditions IE (□), EL (×) and GE (○). The bars indicate ± 1 SEP.

transformed line. Performance was better in these conditions than in the IE condition. Thus vertical disparity had a disruptive effect on discriminability. This agrees with the observers' reports of "instability" and suppression at the endpoints of the lines for some of the stimuli.

Perhaps the most interesting aspect of these data is the general inverted-U shape of all the curves. Clearly, discriminability is worse for elevations near horizontal than it is for elevations equally close to vertical, even though the maximum horizontal disparity h' is far greater for small elevations.

Consider the predictions of both the horizontal meridian model and the nearest neighbor model, under the simplifying assumption that what the observer evaluates in this task, is the maximum disparity at the ends of the lines. Figure 6 shows an oblique line and its vertically magnified counterpart. Suppose that binocular associations of points on the lines in the two half-images are between corresponding points along horizontal meridians (i.e. between points lying on lines parallel to GD). It was shown in the previous paper, that this maximum horizontal meridian disparity h' is equal to

$$h(1 - 1/M)$$

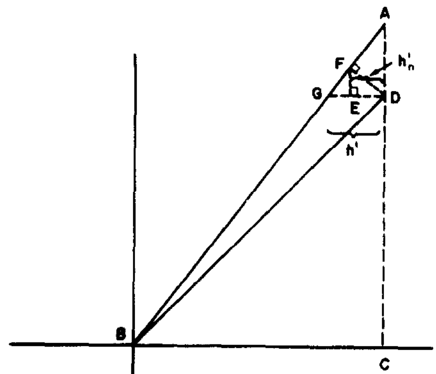

Fig. 6. Diagram illustrating the maximum nearest neighbor disparity for an oblique line (BD) and its vertically magnified counterpart (AB).

where h is the horizontal extent of the line and M is vertical magnification factor.

Now consider the possibility that binocular associations for depth computation are between points which are closest in absolute position (e.g. the segment FD). Also suppose that the disparity which gives rise to the sensation of depth is the horizontal separation of such points (in this case the segment ED). These are "nearest neighbor" disparities*. The Appendix shows that the maximum nearest neighbor disparity h'_n is equal to

$$h'(Mv)^2/[(Mv)^2 + h^2]$$

where M is magnification factor, and v and h are vertical and horizontal extents of the lines. Figure 7 plots h'_n and h' as functions of angle of elevation. A nearest neighbor model predicts an inverted U-shaped discrimination function of elevation, whereas a meridional model predicts a monotonically decreasing discrimination function.

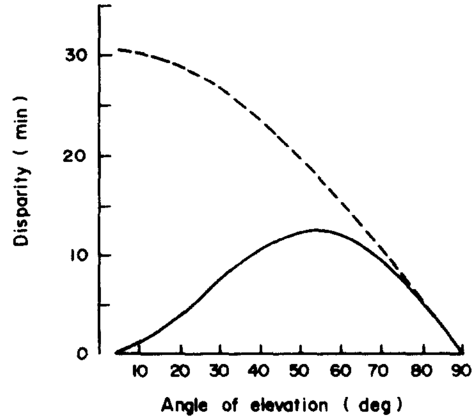

Fig. 7. Graph showing maximum disparities h' (- - -) and h'_n (——) for the stimuli used in Experiment 1 as a function of the elevation θ.

* The nearest neighbor of a point on the unmagnified line lies on a line perpendicular to the magnified line. The nearest neighbor of a point on the magnified, on the other hand, lies on a line perpendicular to the unmagnified line. The horizontal disparities resulting from these two associations are different by minute amounts, and are of no measurable consequence to the model.

Other explanations of these data might rely on the orientation property of the lines rather than the disparity *per se*. For example, Mitchell and O'Hagan (1972) found that the threshold for horizontally disparate vertical lines was smaller than the threshold for horizontally disparate horizontal lines. They hypothesized that vertical lines are inherently superior to horizontal lines in their ability to evoke a stereoscopic sensation. Applying and extending this explanation to the present data, perhaps the ability to evoke stereopsis grows with line elevation. The loss of discrimination for high elevations could then be handled by the fact that the horizontal disparity in these stimuli falls below threshold. The problem with this kind of explanation is that it does not agree well with other observations of differential perceptual responses as a function of orientation. The various "oblique" effects (e.g. Howard and Templeton, 1966; Campbell *et al*., 1966) all show the superiority of vertically and horizontally oriented stimuli over obliquely oriented stimuli to evoke accurate perceptual responses. The present experiment, on the other hand, shows a superiority of oblique elements and, indeed, an inferiority of near horizontally oriented elements.

EXPERIMENT 2

Experiment 1 showed that observers can discriminate the direction of tilt in the induced effect only if oblique lines are contained in the stimulus pattern, and that discriminability depends strongly on elevation of the stimulus lines. This dependence was modeled under the assumption that the discrimination involves evaluating the maximum effective disparity of the stimulus. The data supported the idea that the maximum effective disparities were nearest neighbor disparities (h'_n), rather than disparities along strictly horizontal meridians (h'). Experiment 2 was the analogous experiment on judgements of tilt in the induced effect. Its purpose was to determine whether apparent tilt, too, depends on elevation, and if so, whether the dependence exists over a wide range of vertical magnification.

In this experiment observers were presented with a horizontal adjustment line stimulus in addition to the crossed lines pattern, as shown in Fig. 4b. The apparent tilt of this line was under continuous control of the observer, via a knob, which when rotated, increased the length of the adjustment line in one half-field and reduced the length of the line an equal amount in the other, so that the adjustment line appeared to rotate about a vertical axis. The observer's task was to set the tilt of this line to be apparently coplanar with the plane containing the crossing stimulus lines. After he was satisfied with the adjustment, he terminated the trial by means of a button press. The values recorded were the relative lengths of the adjustment line, expressed as a ratio (hereafter referred to as interocular magnification). It is important to remember that this dependent variable always reflects the *horizontal* interocular magnification in the adjustment line. Since at the beginning of each trial the interocular magnification of the adjustment line was reset to 1, the line also served as an initial reference to define the objective fronto-parallel plane.

Figure 8 plots the adjusted interocular magnifica-

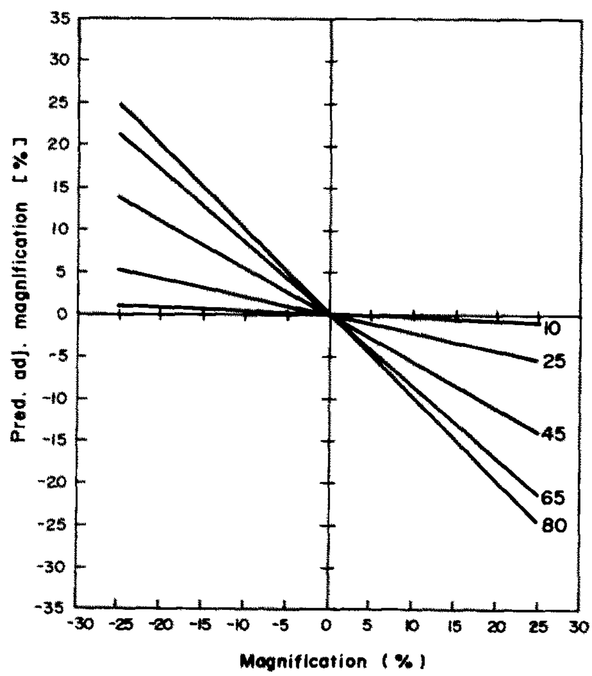

Fig. 8. Adjusted interocular magnification curves predicted by the nearest neighbor hypothesis. The parameter is elevation in degrees. The horizontal meridian hypothesis predicts that all the curves should have a slope of −1.

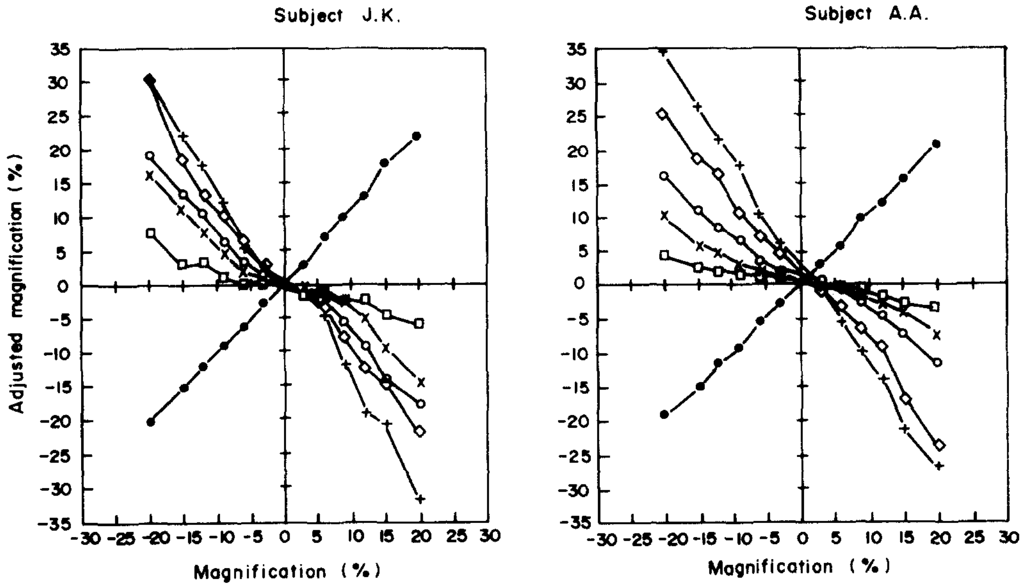

Fig. 9. Adjusted per cent magnifications for 2 observers. Elevations are (□) 10, (×) 25, (○) 45, (◇) 65, and (+) 80 deg. Those spanning the second and fourth quadrants are adjustments to the induced effect stimulus (Experiment 2); those spanning the first and third quadrants are adjustments to the geometric effect stimulus (Experiment 3) for 25 deg elevation.

tion (in units of per cent magnification) between the eyes predicted by the nearest neighbor model. This model would predict that adjusted magnification $h/(h - h'_n)$ should grow not only with vertical magnification, but also with elevation of the stimulus lines. The horizontal meridional model, on the other hand, predicts that adjusted interocular magnification ratio $h/(h - h')$ will grow only with magnification and not with elevation. That is, all under this hypothesis, all the curves should have a slope of -1.

There were five different types of blocks in the experiment, corresponding to the five different angles of elevation (10, 25, 45, 65 and 80 deg) tested. A block consisted of five adjustments at a single orientation for each of twelve randomly ordered levels of vertical magnification (3, 6, 9, 12, 15 and 20%). The five different blocks were initially run in a random order, after which the experiment was repeated, with the order of the blocks reversed, yielding a grand total of 10 adjustments to each stimulus.

Results

Adjusted interocular per cent magnifications are shown spanning the second and fourth quadrants of the graphs in Fig. 9, for vertically magnified oblique lines at the five angles of elevation. Standard errors of these judgements are all quite small, typically less than 5% of the adjusted value, and are omitted for clarity.

It is evident that elevation is a strong determinant of the apparent tilt of such oblique lines. This is confirmed by an F-test for differences in slope of the least squares regression lines (Draper and Smith, 1966) fitted to the data points for each elevation. The hypothesis that the data points arise from the overall regression line plus a random error term is rejected at the 0.01 level of significance for both observers (for observer J.K., $F_{4,50} = 8.70$; for observer A.A., $F_{4,50} = 11.16$).

Thus lines which are near horizontal appeared to be tilted far less than those near vertical. Recall that a model of the visual system in which points are paired along strictly horizontal meridians would predict no effect of elevation; furthermore, it would predict a slope of -1 for all of the curves. These data are at variance with such a model, and at least qualitatively support the nearest neighbor hypothesis.

However, the data in Fig. 9 reveal a quantitative failure of the nearest neighbor prediction: the observers' adjustments are greater, for all elevations, than those predicted by the model in Fig. 8. However, the curves with slopes exceeding -1 (65 and 80 deg) are not well predicted by any geometric model. Thus, it is probable that this failure is due to observers' bias to overestimate the tilt in all the stimuli.

In fact, there is a good reason why such a bias should exist. Recall that the adjustment line was a horizontal line of variable length disparity in the two eyes. Such a configuration alone is less potent as a stimulus to depth in the presence of an inadequate frame of reference (e.g. Wallach and Lindauer, 1962), a phenomenon often referred to as "depth contrast." Now consider the adjustment line with the crossed line stimulus pattern. Due to the vertical magnification, the ends of the crossed line pattern resulted in suppression and rivalry, an observation often noted by the observers. If this instability at the endpoints caused the crossed lines to be a less than adequate frame of reference for seeing depth in the adjustment line, observers may have unwittingly increased the tilt in their adjustments in order to match the apparent tilt of the adjustment line to the apparent tilt of the stimulus lines.

EXPERIMENT 3

In Experiments 1 and 2, elevation proved to have strong effects both on detectability and apparent tilt in the induced effect. The results of Experiment 1 also indicated a dependence on elevation of depth discrimination with oblique lines horizontally magnified in one-half-image. This dependence was explained by the nearest neighbor hypothesis.

Vertical magnification of one half-image, however, is a condition which is rarely encountered outside the laboratory. Thus a dependence of apparent tilt on elevation has little ecological significance. However, a similar dependence on elevation for stimuli horizontally magnified in one half-image would represent a notable departure from veridicality. Experiment 3 explored this possibility.

Except for the fact that the lines in one half-field were horizontally rather than vertically magnified, Experiment 3 was identical to Experiment 2. Note that the predictions of the horizontal meridian and nearest neighbor models are the same as those described above for vertical magnification, but of opposite sign.

Results

Results are shown for two observers spanning the first and third quadrants of Fig. 9, for 25 deg only. The data points for the other angles of elevation are omitted for clarity, since they are all very close to the data for 25 deg. Figure 10 plots the absolute value of the slopes of the regression lines of Experiment 2 and 3, averaged over the two observers.

The F-test described for Experiment 2 was also applied to these data. The hypothesis that the points arise from a common regression line was not rejected

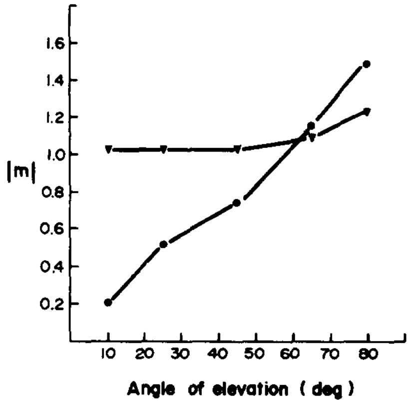

Fig. 10. Summary graph for Experiments 2 and 3, plotting the average absolute values of the slopes and the least squares regression lines fitted to the vertical magnification adjustment (O) and horizontal magnification adjustment (△) curves.

for either observer (for observer J.K., $F_{4,50} = 0.35$; for observer A.A., $F_{4,50} = 0.13$). Thus in contrast to the results of Experiment 2, observers in this experiment perceived the tilt in these displays veridically, in that the adjusted interocular magnifications were more or less constant over all angles of elevation, and with near unit slope. Thus for horizontal magnification of one half-image of oblique line patterns, the horizontal meridian model is a better predictor of apparent tilt.

Another consequence of these results is that the dependence on elevation seen in Experiment 2 cannot be accounted for by a differential bias to see vertical lines as more tilted than horizontal lines. For, such a bias would affect the present results equally.

Also, observers reported that the present task was a great deal easier to perform than that of the previous experiment because of the more stable impression of depth at the endpoints of the crossed lines. This lends additional credence to a "depth contrast" explanation for the overestimation of apparent tilt in the previous experiment.

A major paradox yet to be explained, however, is why the dependence on elevation occurs with vertical but not horizontal magnification of one half-image. The two conditions, after all, are virtually identical except for the presence of vertical disparity of the endpoints in the former condition. A possible reason for this concerns the role of voluntary vergence eye movements. With horizontal magnification, the observer can evaluate the depth by "locking" the endpoint features of the disparate half-images with horizontal eye movements. With vertical magnification this is not possible due to the presence of vertical disparity at the ends of the lines. It seems likely that their veridical performance in the present task is due to the addition of reliable eye movement information.

OTHER DEMONSTRATIONS

Experiment 2 showed that the apparent tilt from fronto-parallel oblique lines vertically magnified in one half-field depends on their obliquity (elevation). Figure 11a and b are stereograms composed of lines near vertical ($\theta = 80$ deg) and lines near horizontal ($\theta = 10$ deg) respectively. The vertical lines in the stereograms are provided as a fronto-parallel referent. The plane defined by the oblique lines of Fig. 11a appears rotated about a vertical axis more than those of 11b.

The stereograms of Figs 12 and 13 show that this difference is not due merely to criterion. Both are constructed with lines at 0, 15, 30, 45, 60, 75 and 90 deg angles of elevation. The left half-image of Fig. 12a is magnified vertically by 20%. When binocularly viewed with crossed eyes, the spokes of this stereogram appear to lie on a surface which bulges and bows in depth very much like the surface represented by the perspective grid of Fig. 12b. In fact, the depth axis values for that grid are, to scale, those which

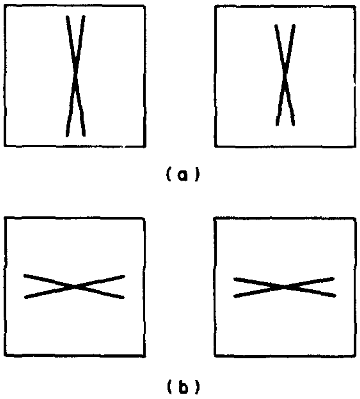

Fig. 11. Induced effect line stereograms in which the left half-image is vertically magnified by 20%. Angles of elevation of the unmagnified lines are (a) 80 deg and (b) 10 deg.

would arise from point-for-point "nearest neighbor" disparities for a disk vertically magnified in one half-image by 1.2. This strongly suggests that, for this stereogram, the nearest neighbor disparity is what determines the tilt.

Figure 13a is identical to 12a, except that the endpoints of the "magnified" half-field have been short-

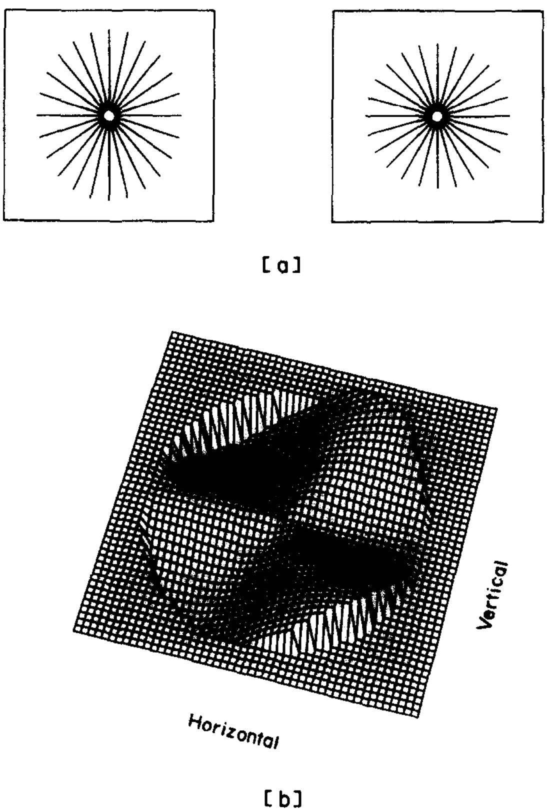

Fig. 12. (a) Induced effect line stereogram demonstrating the dependence of apparent tilt on elevation simultaneously. The left half-image is vertically magnified by 20%. (b) Perspective grid surface portraying a disc whose depth is governed by nearest neighbor disparities for vertical magnification of 20%. Inspection of (a) with crossed eyes yields an apparent surface similar in its depth values to (b).

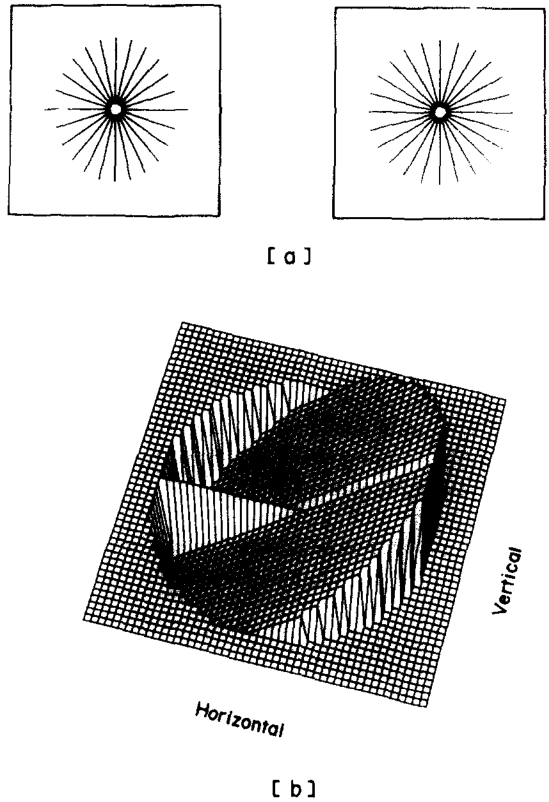

Fig. 13. (a) Same as Fig. 12(a) but with endpoints of the left half-image shortened so as to remove all of the purely vertical disparity. (b) Perspective grid surface portraying a disc whose depth is governed by disparities across horizontal meridians. Inspection of (a) with crossed eyes yields an apparent surface similar in its depth values to (b).

ened so as to remove all of the vertical disparity at the endpoints (analogous to the GE condition of Experiment 1). In other words, the maximum horizontal disparities at the endpoints of this stereogram are those which would arise if the visual system associated points along horizontal meridians. This stereogram appears to be constant tilt except along its major horizontal meridian (where vertical magnification is undefined). The depth values for the perspective grid of Fig. 13b are those which would arise according to a horizontal meridian model. In contrast to Fig. 12 it strongly appears that, in this case, disparity computations are made along horizontal meridians.

Yet the only difference between 12a and 13a is the presence of small endpoint segments which are vertically disparate. It seems that the endpoints of the unmagnified half-image associate with their nearest neighbor in the magnified half-image only when there is vertical disparity between the endpoints, that is, when voluntary vergence cannot resolve the disparity.

GENERAL DISCUSSION

As the theory stands now, a horizontal meridian model of the induced effect does not predict that elevation should affect apparent tilt. Nevertheless, the hypothesis that the induced effect requires oblique lines and their associated horizontal disparities is supported by the results of these two experiments. For Experiment 1, direction of tilt was discriminated only for oblique lines. Furthermore, the present experiment demonstrates that apparent tilt varies with interocular vertical magnification of oblique lines. This result mimics the essential results which Ogle (1938) obtained with meridional lenses.

The dependence of stereoscopic responses on elevation is perhaps more interesting because it poses the question of which points are selected for disparity computation. This type of effect is not without precedent, however. Ebenholtz and Walchli (1965), for example, found that stereoscopic thresholds were proportional to the cosine of the elevation from horizontal for a stimulus consisting of two parallel binocular lines. Figure 14 shows the type of stimulus which they used, for 3 elevations. Since simply varying the elevation of their stimulus does not increase or decrease the maximum horizontal disparity (the distance between the squares and solid circles in Fig. 14) they suggested that the decrease in positional separation (the distance between the filled and open circles) of the lines in the two half-fields which accompanies rotation of the lines, inhibits the effectiveness of the horizontal disparities.

The nearest neighbor hypothesis can explain their observations more parsimoniously. For the nearest neighbor disparities between points on the lines of their stimuli are in fact inversely proportional to the cosine of the elevation. Thresholds, then should be proportional to the cosine of the elevation. Ogle (1955) also fitted the cosine function to other classic

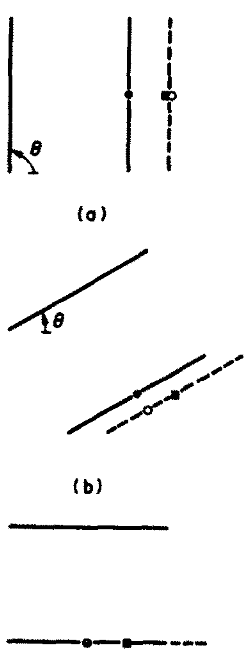

Fig. 14. After Ebenholtz and Walchli (1965). See text.

measures of stereoacuity as a function of line elevation.

An important question alluded to earlier in this paper is why apparent tilt depends so strongly on elevation when one half-image is vertically magnified but not when it is horizontally magnified. The explanation favored here, is that when one half-image is horizontally magnified, vergence movements can resolve the ambiguities in the stimulus since the endpoints of the lines in the two half-fields can fall into perfect binocular registration. This registration may be accompanied by a motor response which Ogle (1950) called the "compulsion to fusion". On the other hand, when one half-image is vertically magnified, voluntary vergence can never allow the endpoints to so resolve. Indeed, Mitchell (1970) showed that the degree of similarity of half-images which is sufficient to initiate vergence eye movement responses is less than that required for completion of those responses. Thus while the similarity of local regions of the lines in both the induced and geometric effects may stimulate vergence eye movements and stereopsis, the completion of those movements may be required to evaluate depth veridically. It is also interesting to note here that Foley and Richards (1972) found perceived depth to grow monotonically with disparity only when free eye movements were allowed, although the failures of monotonicity they observed occurred with disparities much larger than those studied here. The eye movement hypothesis, of course, is readily testable using flashed stimuli, and such experiments will appear in a subsequent paper.

Another unanswered question concerns the range of disparities under which the nearest neighbor hypothesis can be expected to hold. For example, increasing magnification beyond the limits studied here may result in nearest neighbor disparities which are too great to evoke stereopsis. There may also be an orientation disparity limit, beyond which binocular associations cease to occur.

In order to place the hypothesis in proper perspective, it should be noted that a component of it has been an implicit assumption in many investigations of stereopsis with vertical disparities (e.g. Ogle, 1950; Mitchell, 1970). For it has often been correctly assumed that the stereopsis mechanism "tolerates" vertical disparities, rather than responding to them as it does to horizontal disparities. The nearest neighbor hypothesis simply adds the idea that points in the two half-fields are associated by virtue of their proximity, when all other factors, e.g. similarity of features, are equal.

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APPENDIX

Derivation of h'_n

Figure 6 shows an oblique line (the segment BD) and its vertically magnified counterpart (AB). In this figure, h' is the length of segment GD. Arditi *et al.* (1981) previously demonstrated that h', the disparity predicted by the horizontal meridian model, is given by the equation

$$h' = h(1 - 1/M)$$

where h is the horizontal extent of the line, and M is the vertical magnification factor (which may also be expressed as $(v + v')/v$ where v is vertical extent and v' is maximum vertical disparity).

We wish to derive an expression for h'_n, the length of segment ED. Nothing that ABC and FGD are similar triangles,

$$FD/GD = AC/AB = Mv/\sqrt{(Mv)^2 + h^2}$$

therefore,

$$FD = h'Mv/\sqrt{(Mv)^2 + h^2}.$$

Since FDE and ABC are also similar triangles, it is also the case that ED/FD = AC/AB. Therefore,

$$ED = h'_n = h'(Mv)^2/[(Mv)^2 + h^2].$$